Polarizable Vacuum "Metric Engineering" Approach to GR-Type Effects

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ABSTRACT

In addition to the development of the canonical tensor formulation of GR, alternative approaches have emerged over time for investigating metric changes in other formalisms. One such formulation, the polarizable vacuum (PV) approach, derives from a model by Dicke and is related to the *THεμ* formalism used in comparative studies of gravitational theories. The PV approach treats the vacuum as a variable refractive index medium in which vacuum polarizability alters in response to GR-type influences. At one level the PV approach can be characterized as simply a convenient methodology for calculating GR effects in an engineering-type context that provides heuristic insight; at a deeper level the PV formalism can be taken to constitute a fundamental theory in its own right with testable predictions at variance with those of GR under certain conditions. Though in its simplest form PV is a scalar field theory, it has nonetheless been found to be quite general, and to reproduce to required orders a match to several GR effects, including the classical experimental tests of GR. It is in application that the PV formalism demonstrates its intuitive appeal, and constitutes what can be called a "metric engineering" approach. PV studies published to date have addressed not only the standard tests of GR, but also such issues as alteration of the metric by EM fields and corollary implications for spaceflight propulsion. At this stage of our measurement ability, the PV approach appears to be a viable, mathematically simpler alternative to the standard GR approach for investigating variable metric effects. Here we explore the generation and detection of gravitational radiation within the PV theory, noting in particular a predicted longitudinally-polarized mode that might be of significance for HFGW (high-frequency gravitational wave) applications.

1. INTRODUCTION

In the polarizable vacuum (PV) approach to GR effects, the vacuum is treated as a polarizable medium of variable refractive index [1]. Specifically, based on the isomorphism between Maxwell's equations in curved space and a medium with variable refractive index in flat space [2,3], the curved metric is treated in terms of variations in the permittivity and permeability constants of the vacuum, ε_o and μ_o, along the lines of the *"THεμ"* formalism used in comparative studies of alternative gravitational theories [4]. The PV approach, introduced by Wilson [5], developed by Dicke [6,7], and recently elaborated by Puthoff [1], reproduces results predicted by GR for standard (weak-field) astrophysical conditions, and, when considered as a fundamental theory in its own right, poses testable modifications for strong-field conditions [1] and cosmological scenarios [8].

In application the PV approach provides additional insight into what is meant by a curved metric. For example, the bending of a light ray near a massive body is seen as due to an induced spatial variation in the refractive index of the vacuum near the body, the reduction in the velocity of light in a gravitational potential is due to an effective increase in the refractive index, and so forth. This optical-engineering approach has been shown to be quite general and to reproduce to required order various metrics of GR, and thus the match to the associated classical experimental tests of those metrics [9,10].

2. METHODOLOGY

The PV methodology employed here follows that of Ref. [1]. As explained in detail there, the PV treatment of GR effects is based on an action principle (Lagrangian) that holds in special relativity, but with the modification that the velocity of light c in the Lorentz factors and elsewhere is replaced by the velocity of light in a medium of variable refractive index, c/K; expressions such as $E = mc^2$ are still valid, but take into account that $c \to c/K$; and E

$(=E_o/\sqrt{K})$ and m $(=m_o K^{3/2})$ are now functions of K; the vacuum polarization energy associated with the variable K is included, and so forth. The Lagrangian density for a single particle of mass m_0, trajectory $\mathbf{r}(t)$, and velocity $\mathbf{v} \equiv \dot{\mathbf{r}}(t)$, as given by Eqn. (32) of Ref. [1], is

$$\mathscr{L}(\mathbf{r},t) = -\left(\left(m_o K^{3/2}\right)\left(\frac{c}{K}\right)^2 \sqrt{1-\left(\frac{v}{c/K}\right)^2} + q\Phi - q\mathbf{A}.\mathbf{v} \right)\delta^3\left(\mathbf{r}-\mathbf{r}(t)\right) - \frac{1}{2}\left(\frac{B^2}{K\mu_O} - K\varepsilon_O E^2\right) - \frac{\lambda}{K^2}\left[\left(\nabla K\right)^2 - \frac{1}{(c/K)^2}\left(\frac{\partial K}{\partial t}\right)^2\right]. \tag{1}$$

And the resulting Euler-Lagrange particle and field equations that lead to GR-compatible results to testable order as given by Eqns. (33) – (34) of Ref. [1], are

$$\frac{d}{dt}\left[\frac{\left(m_O K^{3/2}\right)\mathbf{v}}{\sqrt{1-\left(\frac{v}{c/K}\right)^2}}\right] = q\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) + \frac{\left(m_o K^{3/2}\right)\left(\frac{c}{K}\right)^2}{\sqrt{1-\left(\frac{v}{c/K}\right)^2}}\left(\frac{1+\left(\frac{v}{c/K}\right)^2}{2}\right)\frac{\nabla K}{K} \tag{2}$$

and

$$\nabla^2\sqrt{K} - \frac{1}{(c/K)^2}\frac{\partial^2\sqrt{K}}{\partial t^2} = -\frac{\sqrt{K}}{4\lambda}\left\{ \begin{array}{l} \dfrac{\left(m_o K^{3/2}\right)\left(\frac{c}{K}\right)^2}{\sqrt{1-\left(\frac{v}{c/K}\right)^2}}\left[\dfrac{1+\left(\frac{v}{c/K}\right)^2}{2}\right]\delta^3\left(\mathbf{r}-\mathbf{r}(t)\right) \\ +\dfrac{1}{2}\left(\dfrac{B^2}{K\mu_O} + K\varepsilon_O E^2\right) - \dfrac{\lambda}{K^2}\left[\left(\nabla K\right)^2 + \dfrac{1}{(c/K)^2}\left(\dfrac{\partial K}{\partial t}\right)^2\right] \end{array} \right\} \tag{3}$$

where $\lambda = c^4/32\pi G$.

We see in (2) that accompanying the usual Lorentz force is an additional dielectric force proportional to the gradient of the vacuum dielectric constant. This term is equally effective with regard to both charged and neutral particles and accounts for the familiar gravitational potential, whether Newtonian in form or taken to higher order to account for GR-type effects. In (3) we see that changes in the vacuum dielectric constant K are driven by mass density (first term), EM energy density (second term), and the vacuum polarization energy density itself (third term). Eqns. (2) and (3), together with Maxwell's equations for propagation in a medium with variable dielectric constant, thus constitute the master equations to be used in discussing general matter-field interactions in a vacuum of variable dielectric constant as employed in the PV formulation.

3. SAMPLE PV CALCULATIONS

Before applying the PV approach to the HFGW case of interest here, we present two calculations that demonstrate the variable refractive index concept in a straightforward manner.

3.1 Bending of Light Rays

The first example is afforded by the bending of a light ray near a body of mass M (e.g., the sun). In the space surrounding the body Eq. (3) takes the form (in spherical coordinates)

$$\frac{d^2\sqrt{K}}{dr^2}+\frac{2}{r}\frac{d\sqrt{K}}{dr}=\frac{1}{\sqrt{K}}\left(\frac{d\sqrt{K}}{dr}\right)^2 \tag{4}$$

where we have used $\left(\nabla K\right)^2 = 4K\left(\nabla\sqrt{K}\right)^2$.

The solution that satisfies the Newtonian limit is given by

$$\sqrt{K}=e^{GM/rc^2}=1+\left(\frac{GM}{rc^2}\right)+\cdots \tag{5}$$

which can be verified by substitution into the equation for particle motion, Eq. (2), for the case $GM/rc^2 << 1$. Since *K* rises in value near the mass, there is a corresponding reduction in the velocity of light $v_L = c/K$ in that region. Therefore, for a light ray grazing the body, that part of the wavefront closest to the body will, by virtue of its reduced velocity, cause the wavefront to tilt such that the light ray is deflected toward the body. This deflection, in GR terms, yields a measure of "local curvature," while in the PV approach it is interpreted as a measure of the spatially dependent vacuum polarizability as represented by *K*.

From Eq. (5) the velocity of light for $GM/rc^2 << 1$ is given by

$$v_L = c/K \approx c\bigg/\left(1+\frac{2GM}{rc^2}\right)\approx c\left(1-\frac{2GM}{rc^2}\right). \tag{6}$$

With the geometry of a grazing ray as shown in Fig. 1(a), Eq. (6) can be written

$$v_L \approx c\left[1-\frac{2GM}{c^2}\frac{1}{\sqrt{\left(R+\delta\right)^2+z^2}}\right]. \tag{7}$$

Therefore, the differential velocity across the ray is (for $\delta << R$)

$$\Delta v_L \approx \frac{2GM}{c}\frac{R\delta}{\left(R^2+z^2\right)^{3/2}}. \tag{8}$$

With reference to Fig. 1(b) – which has a greatly expanded horizontal scale relative to Fig. 1(a) – as the ray travels a distance $dz \approx cdt$, the differential velocity across the ray results in an accumulated wavefront difference Δz given by

$$\Delta z = \Delta v_L dt \approx \frac{2GM}{c^2}\frac{R\delta}{\left(R^2+z^2\right)^{3/2}}dz \tag{9}$$

and an accumulated tilt angle

$$d\alpha \approx \tan\left(d\alpha\right)=\frac{\Delta z}{\delta}\approx\frac{2GM}{c^2}\frac{R}{\left(R^2+z^2\right)^{3/2}}dz\,. \tag{10}$$

Integration over the entire ray path from $z=-\infty$ to $z=+\infty$ yields the standard result

$$\alpha \approx 4GM/Rc^2 \tag{11}$$

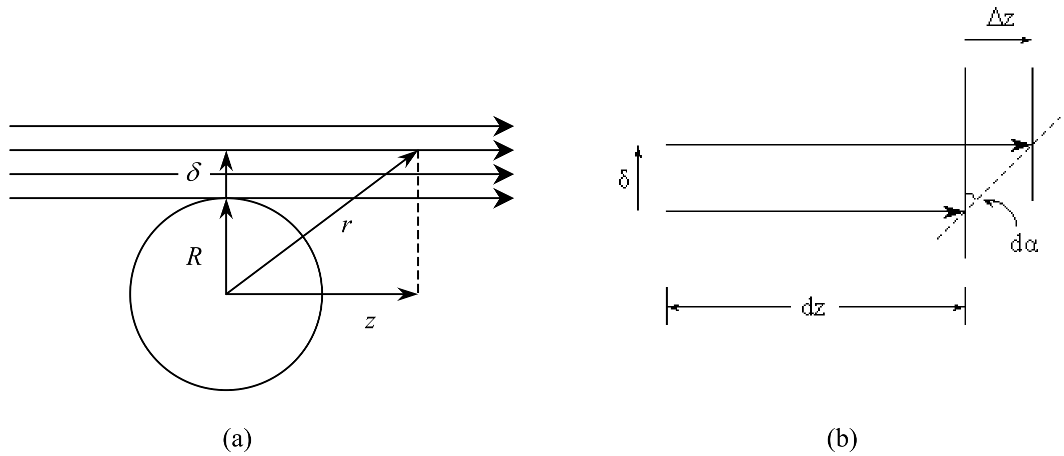


Fig. 1. Geometry of Bending of Light Rays

For other examples of correspondence between PV and GR predictions for standard tests of metric changes in the vicinity of a large mass, we simply note in Table 1 that the weak-field metric tensors for GR (Schwarzschild metric) and PV (exponential metric) are identical to testable order.

Table 1
Metric Tensors

GR: Schwarzschild metric (isotropic coordinates)

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left[\frac{1-\frac{GM}{2rc^2}}{1+\frac{GM}{2rc^2}}\right]c^2dt^2 - \left[1+\frac{GM}{2rc^2}\right]^4\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right)$$

$$g_{00} = 1 - 2\left(\frac{GM}{rc^2}\right) + 2\left(\frac{GM}{rc^2}\right)^2 - ...,$$

$$g_{11} = g_{22} = g_{33} = -\left[1 + 2\left(\frac{GM}{rc^2}\right) + ...\right]$$

PV: Exponential metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{-2GM/rc^2}c^2dt^2 - e^{2GM/rc^2}\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2\right)$$

$$g_{00} = 1 - 2\left(\frac{GM}{rc^2}\right) + 2\left(\frac{GM}{rc^2}\right)^2 - ...,$$

$$g_{11} = g_{22} = g_{33} = -\left[1 + 2\left(\frac{GM}{rc^2}\right) + ...\right]$$

GR vs. PV: Similar argument holds for Reissner-Nordstrøm metric for charged masses.

3.2 Levi-Civita Effect

A second example that demonstrates quite straightforwardly the utility of the variable refractive index approach to spacetime metric distortions is provided by the Levi-Civita effect, the perturbation of the metric by static electric or magnetic fields [11]. Consider the case of a homogeneous, static magnetic field oriented in the z direction (e.g., in the interior of a solenoid of length L, as shown in Fig. 2). Eq. (3) then takes the form

$$\frac{d^2\sqrt{K}}{dz^2} = \frac{1}{\sqrt{K}}\left[\left(\frac{d\sqrt{K}}{dz}\right)^2 - \frac{4\pi G B^2}{\mu_0 c^4}\right] \quad . \tag{12}$$

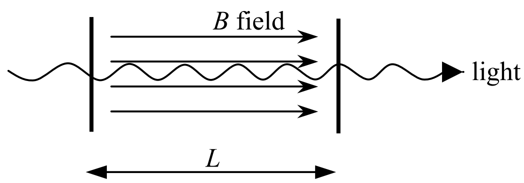


Fig. 2.
Levi-Civita Effect (magnetic field in a solenoid)

The solution to (12) is given by

$$\sqrt{K} = \alpha \cos \beta z \quad , \tag{13}$$

where we have placed the maximum deviation of K from unity at the origin, and the as-yet-undetermined integration constants α and β are found to satisfy the constraint

$$\alpha^2 \beta^2 = \frac{4\pi G B^2}{\mu_0 c^4} \quad . \tag{14}$$

For even the strongest fields of interest (e.g., those of pulsars, $B \sim 10^9$ Tesla) perturbation of the spacetime metric is sufficiently small that Eq. (13) can be approximated by

$$\sqrt{K} \approx \alpha\left(1 - \frac{\beta^2 z^2}{2}\right). \tag{15}$$

We now determine the constants α and β by requiring that the velocity of light $v_L = c / K(z)$ transitions to c (i.e., $K=1$) at a certain distance $L/2$ to the left and right of the origin (e.g., at the ends of a solenoid of length L centered at the origin). With the constants thereby determined, we obtain the solution

$$\sqrt{K} \approx 1 + \frac{2\pi G B^2}{\mu_0 c^4}\left[\left(\frac{L}{2}\right)^2 - z^2\right] \tag{16}$$

and thus the velocity of light $v_L(z)$ within the magnetic field region is given by

$$\frac{v_L(z)}{c} \approx \frac{1}{K} \approx 1 - \frac{4\pi G B^2}{\mu_0 c^4}\left[\left(\frac{L}{2}\right)^2 - z^2\right] \quad . \tag{17}$$

Therefore the velocity of light is slowed within the magnetic field, with its minimum value at the origin, equidistant from the ends of the magnetic field region. The transit time for a light ray through the magnetic field region is thus not L/c but rather

$$\tau \approx \int_{-L/2}^{L/2} \frac{dz}{v_L(z)} \approx \frac{L}{c}\left[1+\frac{2\pi G B^2 L^2}{3\mu_0 c^4}\right] \quad . \tag{18}$$

Similarly, for the electric case,

$$\tau \approx \int_{-L/2}^{L/2} \frac{dz}{v_L(z)} \approx \frac{L}{c}\left[1+\frac{2\pi\varepsilon_0 G E^2 L^2}{3c^4}\right] \quad . \tag{19}$$

Thus the Levi-Civita effect can be understood as a perturbation of the effective refractive index of the vacuum, and summarized in terms of its effect on the propagation of a light ray through the region containing the fields. Again, we see that by use of a relatively transparent PV "metric engineering" formalism, known GR results can be obtained in just a few short steps [12].

4. GRAVITATIONAL RADIATION IN THE PV THEORY

4.1 Overview

To investigate radiation in the PV theory, we begin with Eq. (3), which can be expressed in the form

$$\nabla^2\sqrt{K} - \frac{1}{(c/K)^2}\frac{\partial^2\sqrt{K}}{\partial t^2} = -\frac{\sqrt{K}}{4\lambda}\rho_E \tag{20}$$

where ρ_E is the energy density of mass plus fields. For the weak field regime of interest in the radiation case, $K \approx 1$, we can expand the *K* variable as $\sqrt{K} \approx 1+\phi, \quad \phi \ll 1$, in which case to first order Eq. (20) can be written

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \approx -\frac{8\pi G}{c^4}\rho_E(\mathbf{r},t) \tag{21}$$

with general solution

$$\phi(\mathbf{r},t) = \frac{2G}{c^4}\int d^3\mathbf{r}'\,\frac{\rho_E\left(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c\right)}{|\mathbf{r}-\mathbf{r}'|} . \tag{22}$$

Here we encounter our first major departure from the standard GR approach to gravitational radiation in that we deal with a scalar disturbance that propagates as a longitudinal mode, rather than with a tensor disturbance that propagates as a transverse mode. However, the outcome predictions are surprisingly similar in several particulars, while the differences constitute a testbed for discriminating between the two theories when considered as competitive alternatives.

4.2 Expansion of $|\mathbf{r}-\mathbf{r}'|$ to Different Orders

Expanding $|\mathbf{r}-\mathbf{r}'|$ to second order, we obtain

$$\begin{aligned}|\mathbf{r}-\mathbf{r}'| &= \sqrt{r^2+r'^2-2\mathbf{r}.\mathbf{r}'} = r\sqrt{1+\left(\frac{r'}{r}\right)^2-2\left(\frac{r'}{r}\right)\cos\theta_{\mathbf{r},\mathbf{r}'}} \\ &= r\left(1-\left(\frac{r'}{r}\right)\cos\theta_{\mathbf{r},\mathbf{r}'}+\frac{1}{2}\left(\frac{r'}{r}\right)^2\left(1+\cos^2\theta_{\mathbf{r},\mathbf{r}'}\right)+\ldots\right) . \\ &= r\left(1-\frac{\hat{\mathbf{r}}.\mathbf{r}'}{r}+\frac{r'^2+(\hat{\mathbf{r}}.\mathbf{r}')^2}{2r^2}+\ldots\right)\end{aligned} \tag{23}$$

Therefore the denominator in (22) is

$$\begin{aligned}\frac{1}{|\mathbf{r}-\mathbf{r}'|} &= \frac{1}{r}\left(1+\left(\frac{r'}{r}\right)\cos\theta_{\mathbf{r},\mathbf{r}'}+\left(\frac{r'}{r}\right)^2\cos^2\theta_{\mathbf{r},\mathbf{r}'}-\frac{1}{2}\left(\frac{r'}{r}\right)^2\left(1+\cos^2\theta_{\mathbf{r},\mathbf{r}'}\right)+\ldots\right) \\ &= \frac{1}{r}\left(1+\left(\frac{r'}{r}\right)\cos\theta_{\mathbf{r},\mathbf{r}'}+\frac{1}{2}\left(\frac{r'}{r}\right)^2\sin^2\theta_{\mathbf{r},\mathbf{r}'}+\ldots\right)\end{aligned}. \tag{24}$$

4.3 The Far Field

We are interested in *far-field radiation*, whose characteristic is that the potential falls off as $1/r$. The radiated power is the rate of energy leaving a closed surface some distance from the source. This is computed in turn from the energy density in free space some distance from the source, which is found from the Hamiltonian energy density. The latter is always linear combinations of squares of (space and time derivatives of) the potential. Therefore we need only retain terms in ϕ that fall off as $1/r$, so that the radiated flux falls off as $1/r^2$. Terms in ϕ that fall off faster than this are not deemed radiation because they cannot be regarded as a conserved flux of energy. Consequently we need only consider

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|}\to\frac{1}{r}. \tag{25}$$

With this, (22) is

$$\phi_{rad}(\mathbf{r},t)=\frac{2G}{c^4 r}\int d^3\mathbf{r}'\rho_E\left(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c\right). \tag{26}$$

Expansion of ρ_E according to the last form in (23) yields

$$\begin{aligned}\phi_{rad}(\mathbf{r},t) &= \frac{2G}{c^4 r}\int d^3\mathbf{r}'\rho_E\left(\mathbf{r}',t-\frac{r}{c}\left(1-\frac{\hat{\mathbf{r}}.\mathbf{r}'}{r}+\frac{r'^2+(\hat{\mathbf{r}}.\mathbf{r}')^2}{2r^2}+\ldots\right)\right) \\ &= \frac{2G}{c^4 r}\int d^3\mathbf{r}'\rho_E\left(\mathbf{r}',t-\frac{r}{c}+\frac{\hat{\mathbf{r}}.\mathbf{r}'}{c}-\frac{r'^2+(\hat{\mathbf{r}}.\mathbf{r}')^2}{2rc}\right) \\ &= \frac{2G}{c^4 r}\int d^3\mathbf{r}'\left(1+\left(\hat{\mathbf{r}}.\mathbf{r}'-\frac{r'^2+(\hat{\mathbf{r}}.\mathbf{r}')^2}{2r}\right)\frac{\partial}{c\partial t}+\frac{(\hat{\mathbf{r}}.\mathbf{r}')^2}{2}\frac{\partial^2}{c^2\partial t^2}\right)\rho_E\left(\mathbf{r}',t-r/c\right) \\ \Rightarrow \phi_{rad}(\mathbf{r},t+r/c) &= \frac{2G}{c^4 r}\int d^3\mathbf{r}'\left(1+\hat{\mathbf{r}}.\mathbf{r}'\frac{\partial}{c\partial t}+\frac{(\hat{\mathbf{r}}.\mathbf{r}')^2}{2}\frac{\partial^2}{c^2\partial t^2}\right)\rho_E\left(\mathbf{r}',t\right)\end{aligned} \tag{27}$$

where the $1/r^2$ term has been dropped in accord with the discussion above.

4.4 Contributions From the Different Terms

Zeroth order - monopole

Since

$$\int d^3\mathbf{r}'\rho_E\left(\mathbf{r}',t-r/c\right)=E \tag{28}$$

is a constant, the first term in (27) is constant, and therefore (since radiation depends on derivatives of ϕ) does not contribute to the radiation.

First order - dipole

$\rho_E(\mathbf{r}',t)$ is the density of a conserved quantity, and therefore satisfies

$$\frac{\partial \rho_E(\mathbf{r}',t)}{\partial t} + \nabla'\left(\mathbf{v}'(\mathbf{r}',t)\rho_E(\mathbf{r}',t)\right) = 0 . \tag{29}$$

So the second term in (27) can be written

$$\begin{aligned} \int d^3\mathbf{r}'\hat{\mathbf{r}}\cdot\mathbf{r}'\frac{\partial \rho_E(\mathbf{r}',t)}{\partial t} &= -\int d^3\mathbf{r}'\hat{\mathbf{r}}\cdot\mathbf{r}'\nabla'\cdot\left(\mathbf{v}'(\mathbf{r}',t)\rho_E(\mathbf{r}',t)\right) \\ &= \int d^3\mathbf{r}'\rho_E(\mathbf{r}',t)\mathbf{v}'(\mathbf{r}',t)\cdot\nabla'(\hat{\mathbf{r}}\cdot\mathbf{r}') \\ &= \hat{\mathbf{r}}\cdot\int d^3\mathbf{r}'\mathbf{v}'(\mathbf{r}',t)\rho_E(\mathbf{r}',t) \\ &= c^2\hat{\mathbf{r}}\cdot\mathbf{p} \end{aligned} \tag{30}$$

where **p** is the total momentum of the source. Since **p** is constant, it follows that the second term in (27) is constant in time, and therefore does not contribute to a radiation field. Therefore, only the third term (second order) and higher terms remain to contribute to radiation.

Second order - quadrupole

We have remaining:

$$\phi_{rad}(\mathbf{r},t+r/c) = \frac{G}{c^6 r}\int d^3\mathbf{r}'(\hat{\mathbf{r}}.\mathbf{r}')^2\frac{\partial^2\rho_E(\mathbf{r}',t)}{\partial t^2} . \tag{31}$$

In terms of the Euclidean moment of the inertia tensor

$$\mathbf{I} \equiv \{I_{i,j}\};\quad I_{i,j}(t) = \int d^3\mathbf{r} r_i r_j \rho(\mathbf{r},t) \tag{32}$$

- where the $\{r_i\} \equiv (x,y,z)$ are the Cartesian components of the vector **r**, and where ρ is the *mass* density - (31) can be written

$$\phi_{rad}(\mathbf{r},t) = \frac{G}{c^4 r}\hat{\mathbf{r}}^T\left[\ddot{\mathbf{I}}\right]_{ret}\hat{\mathbf{r}} \tag{33}$$

where the retarded time is $t - r/c$. This can be related to the Einstein GR expression for the weak-field tensor $\bar{\mathbf{h}}$ [13]

$$\phi_{rad}(\mathbf{r},t) = -\frac{1}{2}\hat{\mathbf{r}}^T\left[\bar{\mathbf{h}}\right]_{ret}\hat{\mathbf{r}} . \tag{34}$$

4.5 Radiated power density

The energy density H of the *K*-field is

$$H = \frac{c^4}{32\pi G K^2}\left[(\nabla K)^2 + \frac{1}{(c/K)^2}\left(\frac{\partial K}{\partial t}\right)^2\right] . \tag{35}$$

With the use of the approximation $\sqrt{K} \approx 1+\phi$ this reduces to

$$H(\phi) \approx \frac{c^4}{8\pi G}\left[(\nabla\phi)^2 + \frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right] \tag{36}$$

since the field is weak. At distances far from the source, and provided terms in ϕ are retained only if they fall off as $1/r$, i.e. provided only ϕ_{rad} is retained, then the power per unit area S is the energy density times the speed of light:

$$S_{rad,PV} = cH(\phi_{rad}) = \frac{c^5}{8\pi G}\left[(\nabla\phi_{rad})^2 + \frac{1}{c^2}\left(\frac{\partial\phi_{rad}}{\partial t}\right)^2\right]. \tag{37}$$

With reference to (33), the gradient of ϕ will introduce additional powers of *1/r* when it operates on the $\hat{\mathbf{r}}$, and therefore does not contribute to the power through a surface at a large distance from the source. The spatial gradient operating on the retarded time does contribute, however:

$$\left(\nabla f(t-r/c)\right)^2 = \frac{1}{c^2}\left(\frac{\partial f(t-r/c)}{\partial t}\right)^2 \tag{38}$$

and therefore (37) gives

$$S_{rad,PV} = \frac{c^3}{4\pi G}\left(\frac{\partial\phi_{rad}}{\partial t}\right)^2. \tag{39}$$

Putting (33) into (39) gives

$$S_{rad,PV} = \frac{c^3}{4\pi G}\left(\frac{G}{c^4 r}\right)^2\left(\hat{\mathbf{r}}^T\left[\ddot{\mathbf{I}}\right]_{ret}\hat{\mathbf{r}}\right)^2 = \frac{G}{4\pi r^2 c^5}\left(\hat{\mathbf{r}}^T\left[\ddot{\mathbf{I}}\right]_{ret}\hat{\mathbf{r}}\right)^2. \tag{40}$$

4.6 Radiated Power

From (40), the total power crossing a sphere at radius r is

$$P_{rad,PV} = \frac{G}{4\pi c^5}\oint\!\!\oint d\Omega_{\hat{\mathbf{r}}}\left(\hat{\mathbf{r}}^T\left[\ddot{\mathbf{I}}\right]_{ret}\hat{\mathbf{r}}\right)^2. \tag{41}$$

Conversion to a rotated coordinate system in which $\mathbf{I}$ is diagonal, followed by the application of symmetry principles, can be shown to lead to the integrated result

$$P_{rad,PV} = \frac{G}{15c^5}\left[2Tr\left(\ddot{\mathbf{I}}^2\right) + \left(Tr\left(\ddot{\mathbf{I}}\right)\right)^2\right]_{ret}. \tag{42}$$

For comparison, the GR result (accounting for a factor 3 in the definition of $\mathbf{I}$) is [14]:

$$P_{rad,GR} = \frac{G}{5c^5}\left[Tr\left(\dddot{\tilde{\mathbf{I}}}\right)^2\right]_{ret} \tag{43}$$

where $\tilde{\mathbf{I}}$ is the *reduced* quadrupole moment tensor

$$\tilde{\mathbf{I}} \equiv \mathbf{I} - \frac{1}{3}Tr(\mathbf{I}). \tag{44}$$

Since

$$Tr\left(\dddot{\tilde{\mathbf{I}}}^2\right) = Tr\left(\left(\dddot{\mathbf{I}} - \frac{\mathbf{1}}{3}Tr\left(\dddot{\mathbf{I}}\right)\right)^2\right) = Tr\left(\dddot{\mathbf{I}}^2 - 2\frac{\dddot{\mathbf{I}}}{3}Tr\left(\dddot{\mathbf{I}}\right) + \frac{\mathbf{1}}{9}\left(Tr\left(\dddot{\mathbf{I}}\right)\right)^2\right) = Tr\left(\dddot{\mathbf{I}}^2\right) - \frac{1}{3}\left(Tr\left(\dddot{\mathbf{I}}\right)\right)^2 \tag{45}$$

then (43) can be written

$$P_{rad,GR} = \frac{G}{15c^5}\left[3Tr\left(\dddot{\mathbf{I}}^2\right) - \left(Tr\left(\dddot{\mathbf{I}}\right)\right)^2\right]_{ret}. \tag{46}$$

Comparison of Eqns. (42) and (46) indicates that, despite the apparent differences between the PV scalar approach versus the GR tensor approach, the magnitudes and inertial moment dependencies of the radiated powers are quite similar.

5. ROTATING DUMBBELL EXAMPLE

We consider here a rotating dumbbell to illustrate the similarities and differences between the PV and GR approaches to gravitational radiation. Consider 2 equal masses m, separated by a, rotating about their center of mass in the x-y plane. The mass density is given by

$$\rho = m\delta(z)\left[\delta\left(x - \frac{a}{2}\cos\omega t\right)\delta\left(y - \frac{a}{2}\sin\omega t\right) + \delta\left(x + \frac{a}{2}\cos\omega t\right)\delta\left(y + \frac{a}{2}\sin\omega t\right)\right]. \tag{47}$$

According to (32) this leads to

$$\left[\dddot{\mathbf{I}}\right]_{ret} = 2ma^2\omega^3\begin{pmatrix} \sin\left(2\omega(t-r/c)\right) & -\cos\left(2\omega(t-r/c)\right) & 0 \\ -\cos\left(2\omega(t-r/c)\right) & -\sin\left(2\omega(t-r/c)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{48}$$

and

$$\hat{\mathbf{r}}^T\left[\dddot{\mathbf{I}}\right]_{ret}\hat{\mathbf{r}} = 2ma^2\omega^3\sin^2\theta\sin\left(2\omega(t-r/c) - 2\varphi\right). \tag{49}$$

With this result in hand Eq. (40) yields the power density as

$$S_{rad,PV} = \frac{Gm^2a^4\omega^6}{\pi r^2c^5}\sin^4\theta\sin^2\left(2\omega(t-r/c) - 2\varphi\right) \tag{50}$$

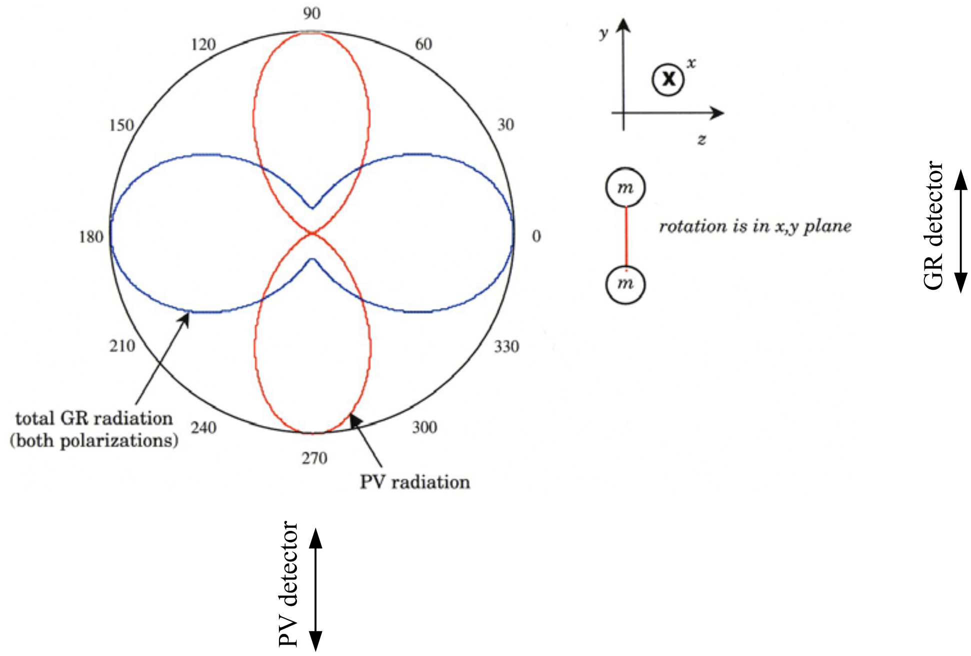
and its time-averaged value

$$\overline{S_{rad,PV}} = \frac{Gm^2a^4\omega^6}{2\pi r^2c^5}\sin^4\theta. \tag{51}$$

This power density is maximal in the x-y plane (the plane of dumbbell motion), and zero in the perpendicular z-direction (see plot in Fig. 3). This is not unexpected, given the longitudinal mode nature of the propagation of the scalar potential. This is in contrast with the GR result [15]

$$\overline{S_{rad,GR}} = \frac{Gm^2a^4\omega^6}{8\pi r^2c^5}\left(1 + 6\cos^2\theta + \cos^4\theta\right). \tag{52}$$

where the power density is maximal in the z-direction perpendicular to the x-y plane and, though not zero, is considerably less in the x-y plane (Fig. 3).



- GR: Maximum intensity is *perpendicular* to plane of rotation.
- PV: Maximum intensity is *in* plane of rotation.
- In both cases, detectors are oriented parallel to plane of rotation.

Fig. 3
Radiation Pattern of a Rotating Dumbbell

As a result of the contrasting distributions for the PV and GR cases, optimal detector placement and orientation differ as shown in Fig. 3. Both are oriented parallel to the plane of dumbbell rotation, but differ in placement. On the basis of the PV-predicted distribution the detector should be placed in the plane of dumbbell rotation to detect a longitudinally-polarized signal; for the GR-predicted distribution the detector should be placed on a line perpendicular to the plane of dumbbell rotation to detect a transverse-polarized signal. Thus experiment can be used to discriminate between the alternative PV and GR theories of gravitational wave generation and detection.

The associated integrated powers for the PV and GR cases are obtained by integration of Eqns. (51) and (52), or evaluation of (42) and (46), to yield

$$P_{rad,PV} = \frac{16Gm^2a^4\omega^6}{15c^5} \tag{53}$$

and

$$P_{rad,GR} = \frac{8Gm^2a^4\omega^6}{5c^5} = \frac{3}{2}P_{rad,PV} \,. \tag{54}$$

Given the mathematically significant difference between the scalar and tensor characters of PV and GR, respectively, such close agreement is quite remarkable.

As a special case, for two equal masses circling a distance a apart and held together by gravitational force, one has $\omega^2 = 2Gm/a^3$ and therefore Eqns. (53) and (54) reduce to forms that can be compared with the literature for this simple gravitationally bound binary case [16],

$$P_{rad,PV} = \frac{128G^4}{15}\left(\frac{m}{ac}\right)^5 \tag{55}$$

$$P_{rad,GR} = \frac{64G^4}{5}\left(\frac{m}{ac}\right)^5. \tag{56}$$

6. CONCLUSIONS

In this paper we have examined an alternative to the standard GR approach to gravitational radiation. Known as the *polarizable vacuum* (PV) or *metric engineering* approach, the formalism treats the vacuum as a variable refractive index medium. The PV formulation can be seen as simply a convenient methodology for calculating GR effects, or be considered a fundamental theory in its own right that addresses spacetime metric variation by alternate means. The PV approach employs a scalar (vs. tensor) mathematical structure that is simple to apply, viable for many applications, and lends itself to engineering analysis and interpretation.

For HFGW (high-frequency gravitational wave) applications, the PV approach explores the possibility of launching a longitudinally-polarized mode, which stands in contrast to the canonical GR treatment in terms of transverse-polarized waves. A corollary of this result is that, though essentially in agreement with standard GR predictions of signal strength for similar apparatus configurations, the PV approach specifies alternative detector placement and orientation for optimal performance. Thus experiment can serve as a discriminator between the two approaches seen as competing fundamental theories.

Finally, in contrast to the GR treatment of metric changes in terms of generalized spacetime distortions, the PV approach treats metric changes specifically in terms of vacuum dielectric perturbations. This suggests a program of exploration of additional HFGW detection schemes specifically designed to detect refractive index variations.

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